The Physics of Low-Altitude Aerobatic Flying

By: Eric Boyd

SPH 3U

Mr. Leck

Submitted: December 19, 2008

Introduction

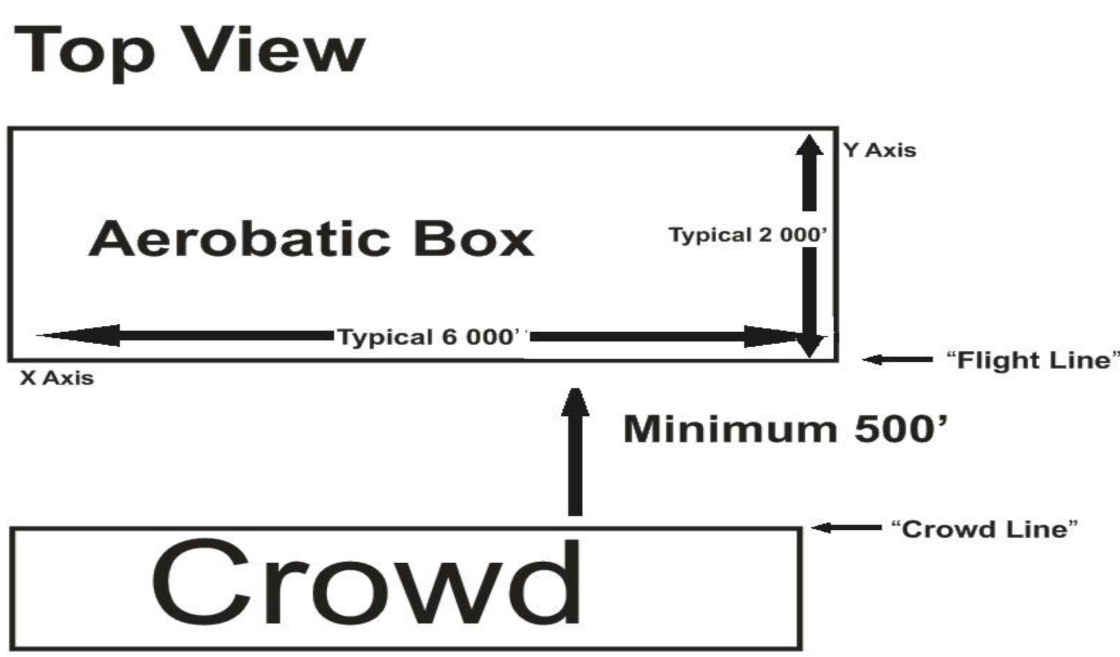
For over half a century, large amounts of the public have come to watch a series of various aerial demonstrations, known as air shows. Various aspects of air shows, especially an aircraft flying low level aerobatics is one of many great applications of physics. Using physics we will learn about the kind of calculations and considerations required to make an aerobatic sequence.

Note: All calculations and examples are in Imperial units. That is, Earth's gravitational field is not thought of as 9.8 meters per second squared, but 32 feet per second squared, and other things of that nature. This is because aircraft measure their airspeed in miles per hour, and constantly converting measurements to metric, rather then just do calculations is Imperial, is timely.

The Environment:

As specified by Canadian Aviation Regulation 623.07, and Federal Aviation Regulation (USA) 8900.1, volume 3, chapter 6, at a public air show, there is a specified three dimensional box in which aircraft are permitted to perform aerobatic manoeuvres.

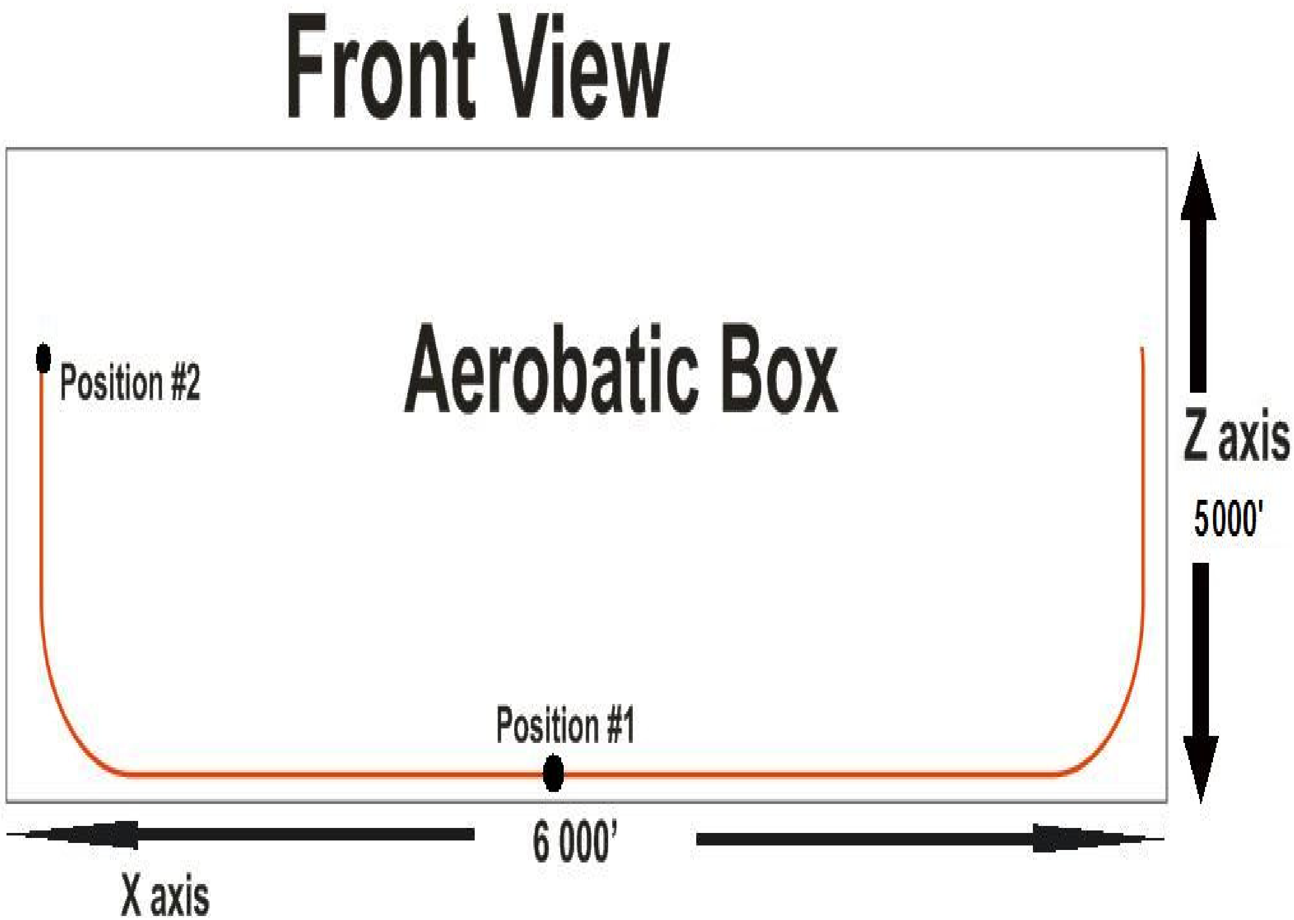
Figure 1



For the safety of the crowd, no aerobatics may be flown over the crowd, only inside the **aerobatic box**. Again, for the safety of the crowd, there are also limitations on the manoeuvres which can be performed in the box.

Almost every aerobatic box is of different dimensions, depending on the aircraft flying, and local considerations. Logically, faster aircraft require larger boxes, and only aircraft participating in the aerobatics and/or aerial demonstration(s) at that current time are permitted in the box, so the size of the box is often restricted by houses, roads, etc. near the airport in which the air show is being held.

Figure 2



The above sample box (Figure 1 and 2) is 6 000 feet long (X axis), 2 000 feet deep (Y axis) and 5 000 feet high (Z axis), typical dimensions for an air show of propeller aircraft only – faster jets require much more room, and hence, larger boxes.

Most of the time, the aircraft at an air show will fly back and forth along the flight line (see Fig. 1),, as to be as close as legally possible to the crowd, as to present a better show, at high airspeed and low altitude. Its trajectory can be (roughly) being seen in Fig. 2 (red line).

When the aircraft approaches the end of the aerobatic box, it will typically perform some variation of a **vertical reversal**. A **horizontal reversal** can be performed, but a vertical reversal is more dramatic and unusual, and thus preferred – crowds come to air shows to see unusual flying, after all.

At the top of the vertical reversal (See position 2, Fig. 2), the aircraft will often be nearly stopped – that is, an **airspeed** of nearly zero. One advantage of the nearly zero airspeed at the top of a vertical reversal is that the radius of the reversal is much smaller than at high speed (we learn more about this in the "Top Gate" section).

Calculating Aircraft's Energy (and applications of such)

At any time, we can calculate the **total mechanical energy** of the aircraft as the total of the **kinetic energy** (due to airspeed) and **gravitational potential energy** (due to height):

$$E_t = E_k + E_g$$

$$E_t = \frac{1}{2} mv^2 + mgh$$

At position 1 (low height and fast speed) (in Fig. 2), we can calculate the total mechanical energy as:

$$E_{t1} = E_{k1} + E_{g1}$$

$$E_{t1} = \frac{1}{2} mv_1^2 + mgh_1$$

Notice that m and g were not given subscripts indicating position 1. This is because mass and gravity are assumed constant (mildly untrue for mass – as the aircraft runs, it consumes fuel, oil, etc thereby slightly reducing its mass.)

Since the altitude of the aircraft is so small, gravitational potential energy can be assumed as 0 through the **property of 0** (anything multiplied by zero, is zero). This leaves us with:

$$E_{t1} = \frac{1}{2} mv_1^2$$

Which is almost completely kinetic energy.

At position 1 in (Fig. 2), we can calculate the total mechanical energy as:

$$E_{t2} = E_{k2} + E_{g2}$$

$$E_{t2} = \frac{1}{2} mv_2^2 + mgh_2$$

Since the airspeed of the aircraft is so small (as previously stated), kinetic energy can be assumed as 0 through the property of 0. This leaves us with:

$$E_{t2} = mgh_2$$

Which is almost completely gravitational potential energy.

So, we can see that an air show aircraft, as it flies back and forth in front of the crowd, is continually converting potential energy to kinetic energy, and vice versa.

If we make the somewhat simplistic assumption that the air show aircraft has the same total mechanical energy (not true – the engine and propeller are constantly trying to replace energy lost to aerodynamic drag, especially induced drag, and we have previously proven that mass is decreasing), we could postulate that

$$\frac{1}{2} m v_1^2 = m g h_2$$

Where v_1 is the velocity at position 1, and h_2 is the height at position 2. Mass then cancels out of the equation, to leave us with:

$$\frac{1}{2} v_1^2 = g h_2$$

$$\therefore \Delta h = \frac{v^2}{2g}$$

From this equation, we can determine how much **vertical penetration** an aircraft has at any given time, using just the velocity of the aircraft.

Also from this equation, we can state that:

$$\Delta h \propto v^2$$

$$\Delta h \propto \frac{1}{2g}$$

Energy Gaining VS Energy Losing Manoeuvres

Logically, at the end aerobatic manoeuvres, the aircraft can have less total mechanical energy then at the start, the same amount of energy, or a greater amount of energy. From the total mechanical energy formula, we know that kinetic and potential contribute to the total energy of the aircraft, so we know that an aircraft can gain energy in a manoeuvre by gaining altitude, or gaining airspeed. So, we know the aircraft has gained energy, and hence, the manoeuvre was an **energy gaining manoeuvre** when

$$E_{k\ gained} > E_{p\ lost}$$,or

$$E_{p\ gained} > E_{k\ lost}$$

So, we also know that the aircraft has lost energy, and hence, the manoeuvre was an **energy losing manoeuvre** when

$$E_{k\ gained} < E_{p\ lost}$$,or

$$E_{p\ gained} < E_{k\ lost}$$

Let's look at an aircraft, which is high, and slow (Position 2, Figure 2), which flies down to being low and fast (Position 1). At that point, the aircraft has converted a significant amount of its potential energy to kinetic but it's total mechanical energy is relatively the same - very high.

Now, lets say the aircraft executes the following manoeuvre: a flat ten "G" level turn, just above the surface, all the way around 360 degrees, ending back at position 1. The airspeed of the aircraft will have decreased significantly, because of the enormous drag produced by the aircraft in the turn (this topic is expanded and explained in the next section – Four Forces Acting on an aircraft), and hence, will have much less total mechanical energy then at it's starting position.

This manoeuvre is never performed by propeller powered aircraft, because their piston engines simply don't produce enough power to generate enough thrust to overcome the drag produced in the turn (again, this topic is explained and expanded on in the next section), and hence will likely stall and crash into the ground. Only military jet aircraft can perform this manoeuvre, because their very powerful afterburning engines can overcome the drag produced, and hence, do not lose significant amounts of energy.

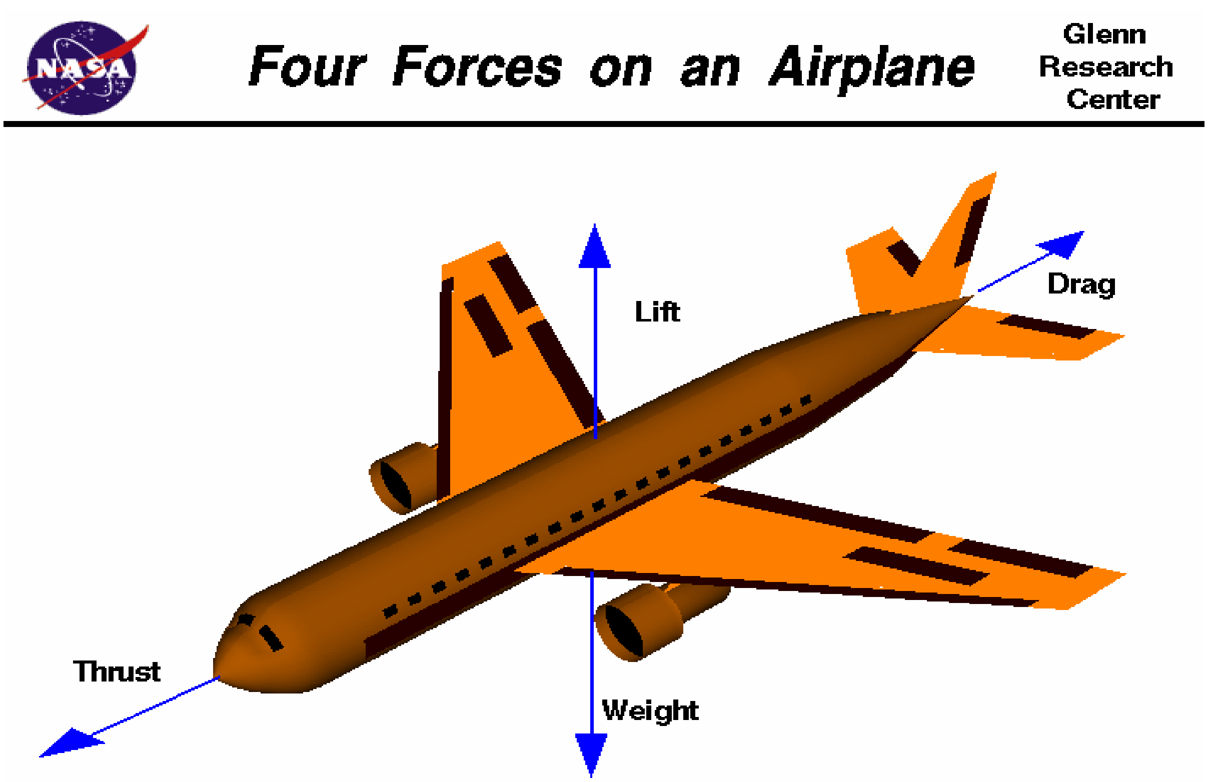
We can see that high G manoeuvres are "Energy losers" because the enormous drag produced in the manoeuvre is constantly consuming energy which cannot be replaced under such powerful drag.

Back to the aircraft which has just performed the flat ten G level turn, and is at position 1, and has little altitude, or airspeed to convert to one another for another manoeuvre. The only option available to the aircraft is to fly away slowly, at a low, but positive angle of attack, as to gain altitude slowly, with a slow velocity, as to not generate much drag (this concept is called the coefficient of drag, and is expanded upon in the next section). Now, the aircraft cannot fly for miles away from position 1, as he wants to remain in the aerobatic box, and as to not bore the crowd for extended period of time. So, the aircraft turns slowly, and has much altitude, and hence potential energy to perform the next manoeuvre.

This manoeuvre is called a "wingover", and is a timely process to reposition the aircraft at the same position with more energy.

The Four Forces Acting on an Aircraft

There are four main external forces acting upon an aircraft at any time in flight. They are **lift**, **weight**, **drag**, and **thrust**.



Logically, if the lift vector is greater then the weight vector, the aircraft gains altitude, and hence potential energy. And if the thrust vector is greater then the drag vector, the aircraft gains airspeed, and hence kinetic energy. The reverse is true for both of the preceding statements.

In order to calculate F_{net}, the net force acting on the aircraft, we must know all 4 vectors:

Lift

We can calculate lift from the lift equation:

$$\mathbf{F_{lift} = (K)\,(C_L)\,(v^2)}$$

Where K is a constant, relating to the air pressure/density and the aircraft's wing area, C_L is the coefficient of lift and is a function of the aircrafts angle of attack, and v is simply the velocity. The coefficient of lift is best viewed and understood as a graph, of the coefficient of lift VS AOA (Angle of Attack):

Weight

The weight can simply be calculated as the **normal force**, or F_n.

$$\mathbf{F_{weight} = mg}$$

Where m is the mass of the aircraft, and g is the acceleration due to Earth's gravity. This can be calculated as

g = (G Forces) (32 feet per second squared)

Knowing the lift and the weight vectors, we can determine the **vertical component** of F_{net}.

Thrust

Eg. Assume a propeller powered aircraft with 300 HP will go 200mph on full throttle, if we convert that to feet per second:

$$\mathbf{200 \times \frac{5280}{3600} \approx 300 \text{ feet per second}}$$

Now 1HP = 550 lbs/foot/second, therefore

300 HP = 550 lbs/foot/second X 300

If we divide that by 300 feet/second (to cancel out the units) we get:

$$\mathbf{300 \times \frac{550}{300} = 550 \text{ Lbs of thrust}}$$

Produced by the propeller. Converting that to Newtons, by multiplying it by the conversion factor, we get:

550 X 4.44822162 ≈ 2 450 Newtons.

Keep in mind that the propeller is not 100% efficient, it "slips" about 10%, varying with the density, shape and position of the propeller.

While we have not perfectly calculated thrust, we have gotten a very good feel for it, at full throttle. Actually calculating thrust precisely is very complicated, and is best explained in **Aircraft Engine Design** by Jack D. Mattingly, William H. Heiser, and Davit T. Pratt, specifically Appendix E. A link to a PDF of the book is included in the bibliography.

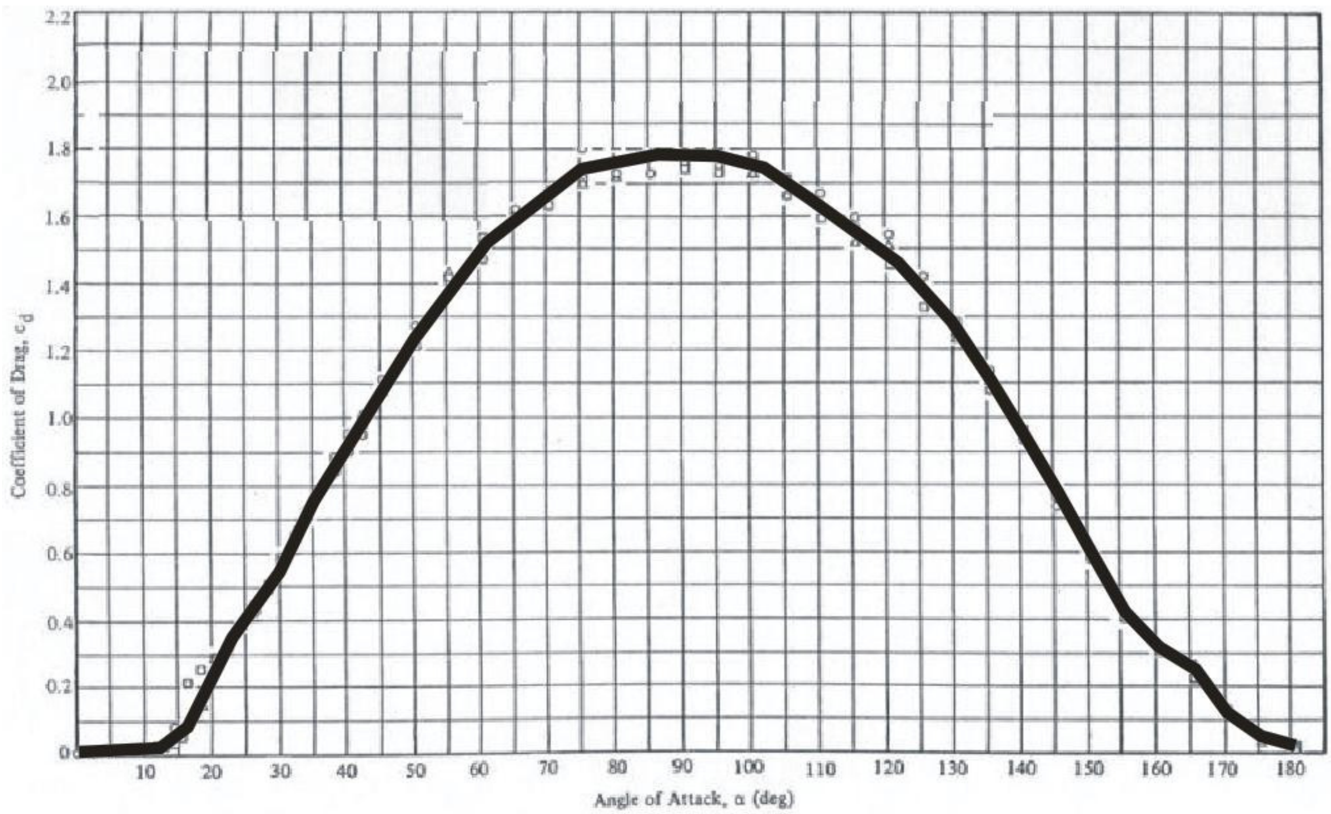
Drag

The equation for drag is:

$$F_{drag} = \frac{1}{2} p V^2 S C_D$$

Where p is the density of the air, V is the velocity, S is the equivalent front plate area (surface area of the aircraft exposed to the air), and C_D is the coefficient of drag.

Like the coefficient of lift, the coefficient of drag is a function of the aircrafts angle of attack, as well as the shape of the aircraft. Again, similar to the coefficient of lift, the coefficient of drag is best interpreted as a graph, of C_d VS AOA:



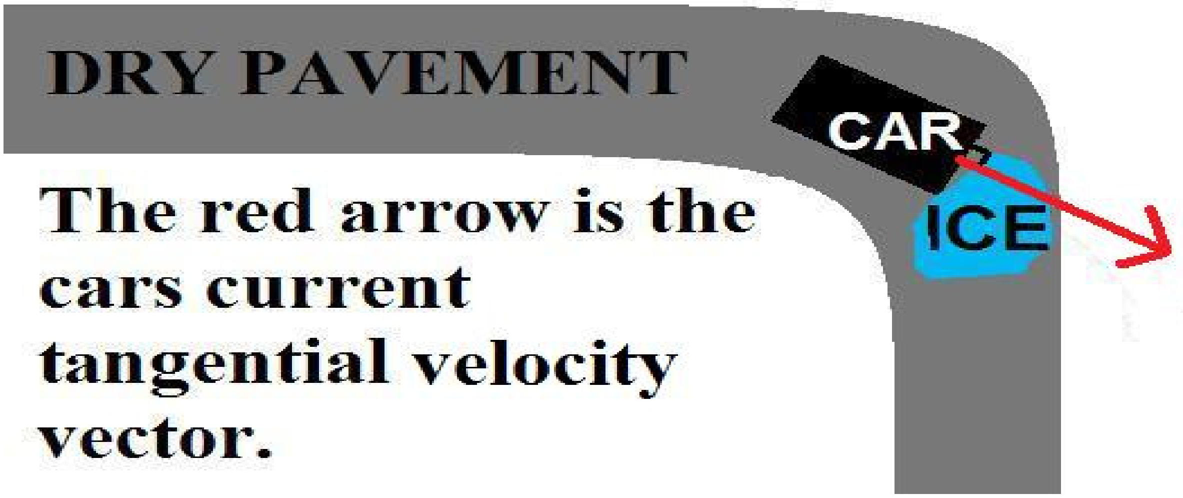
Notice how the coefficient of drag is fairly symmetrical, and C_{dmax} occurs at 90 degrees, which makes sense; at 90 degrees, the aircraft is vertical, and has the most surface area exposed.

Tangential Velocity Vectors

Up to this point, we have been simply considering velocity as a **scalar value**, that is, it has a magnitude, but no direction. This is clearly not the case. A velocity **vector** has a magnitude and direction – eg 10mph [N], which leads nicely into the concept of **tangential velocity vectors**.

The simplest way to understand the concept of tangential velocity vectors is to visualize a car going around a corner:

Figure 3



Once the car encounters the patch of ice, its **lateral acceleration** (which was pulling it around the corner, due to the rubber tires on the dry pavement) ceases, due to the friction dramatically dropping to significantly due to the **coefficient of friction** of rubber on ice being much smaller then rubber on dry pavement. The car then slides into the ditch, because its tangential velocity vector was pointing in that direction at the moment it encountered the ice, and Newton's second law (inertia) states that any object in motion will remain in motion.

The direction the car is going, and the speed in which it is going, determines the tangential velocity vector of the car at any point in time.

Applications of Tangential Velocity Vectors to Aerobatic Manoeuvres

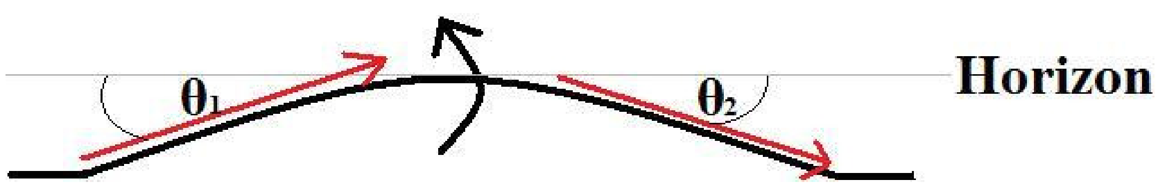
Tangential velocity vectors can be applied to aerobatic manoeuvres to determine how dangerous said manoeuvre is. This is done by calculating the angle of the vectors in the manoeuvre in relation to the horizon (0 degrees). The greatest negative angle contained within the manoeuvre, the more dangerous it is classified. For the purposes of classification, we can coin a term for this angle; logically the "**critical tangential velocity vector angle**" is a good fit.

Safest:

The safest low level aerobatic manoeuvres are rolls (specifically ballistic (AKA Aileron) and slow) and half cuban eights. This is because the rolls are **horizontal aerobatics**, that is, they require little to no change in altitude required to complete the manoeuvre. The half cuban eight is also classified as safest due to it's critical tangential velocity vector being so close to the horizon (0 degrees), as opposed to other **vertical aerobatics**.

Ballistic (AKA Aileron) Roll

The aircraft is **pitched** up (to exit the roll at the same altitude) using the **yoke**, **ascends**, then rolled 360 degrees while ascending, then **descends** with the **wings level**. Notice how θ_1 is equal to θ_2, and how small they are. The C.T.V.V.A. is very small, thus making the ballistic roll one of the safest manoeuvres.

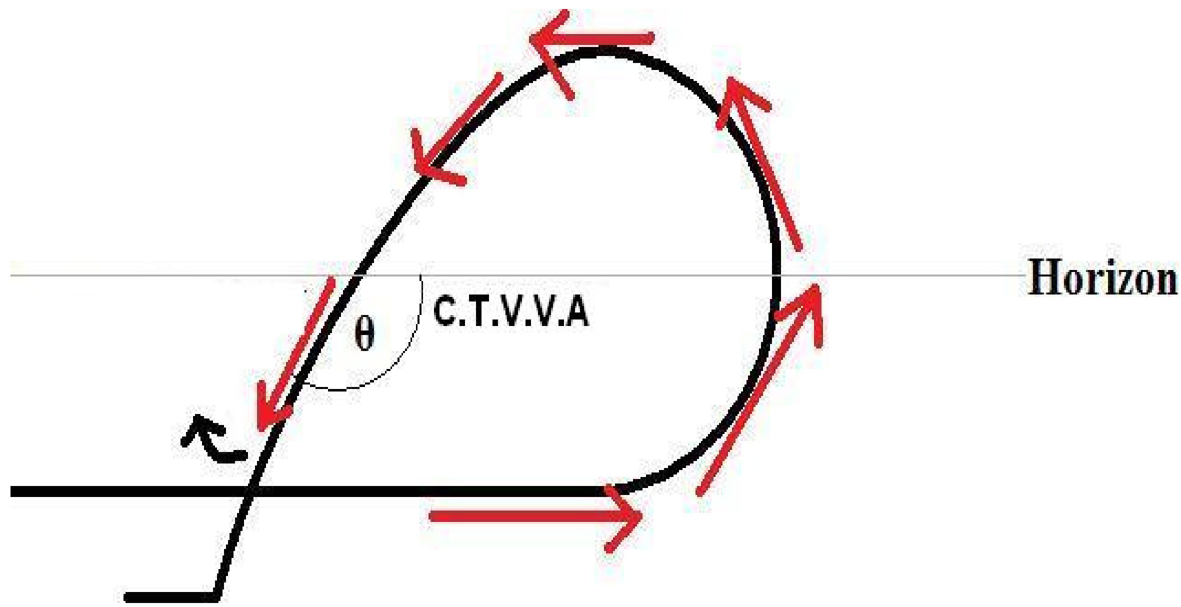


Slow Roll

In level flight, the aircraft is sharply rolled 360 degrees. When a roll of 90 degrees is achieved, **top rudder** is applied to create lift using the fuselage. When a roll of 180 degrees is achieved, the **yoke** is pushed forward, to create lift. When a roll of 270 degrees is achieved, top rudder is applied to create lift using the fuselage. If done properly, no change in altitude should occur, thus your tangential velocity vector is always parallel to the horizon, and θ = 0 throughout, thus the C.T.V.V.A. is 0. This makes the slow roll a very safe manoeuvre.

Half Cuban Eight

After a few moments level flight, the yoke comes back to pitch the aircraft and to continue to pitch the aircraft up , the aircraft continues to pitch through what seems like a **loop**, but at the C.T.V.V.A., a **half roll** (roll 180 degrees) occurs to keep the aircraft under **positive G**. The aircraft then pitches up, and exits the manoeuvre. The C.T.V.V.A. can range from 25 degrees to 65 degrees, depending on how the manoeuvre is flown.

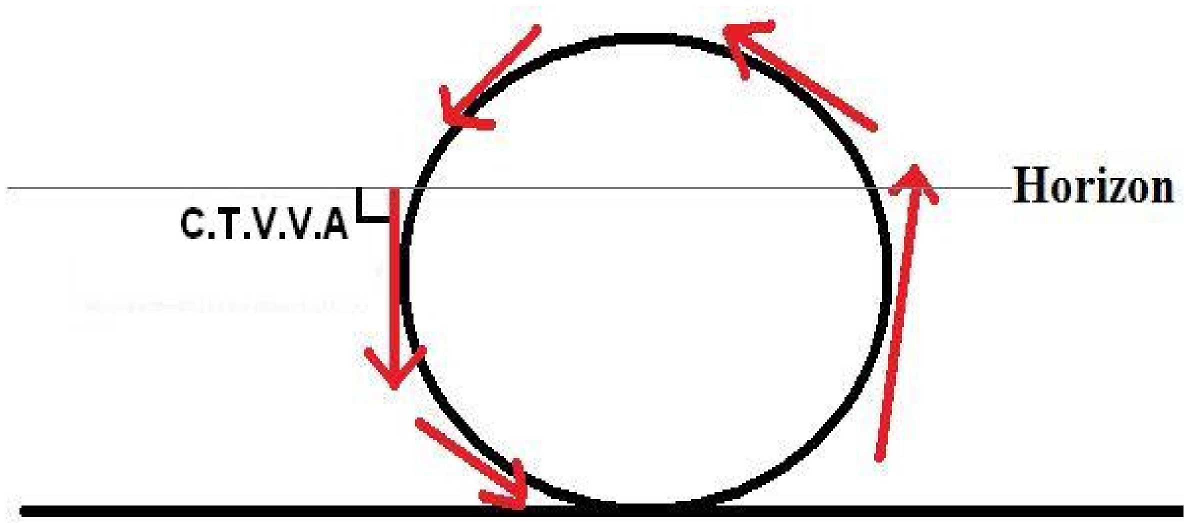


Moderately Dangerous:

Moderately dangerous manoeuvres are classified as such due to the 90 degree – straight down – critical tangential velocity vector angle that occurs during the manoeuvre.

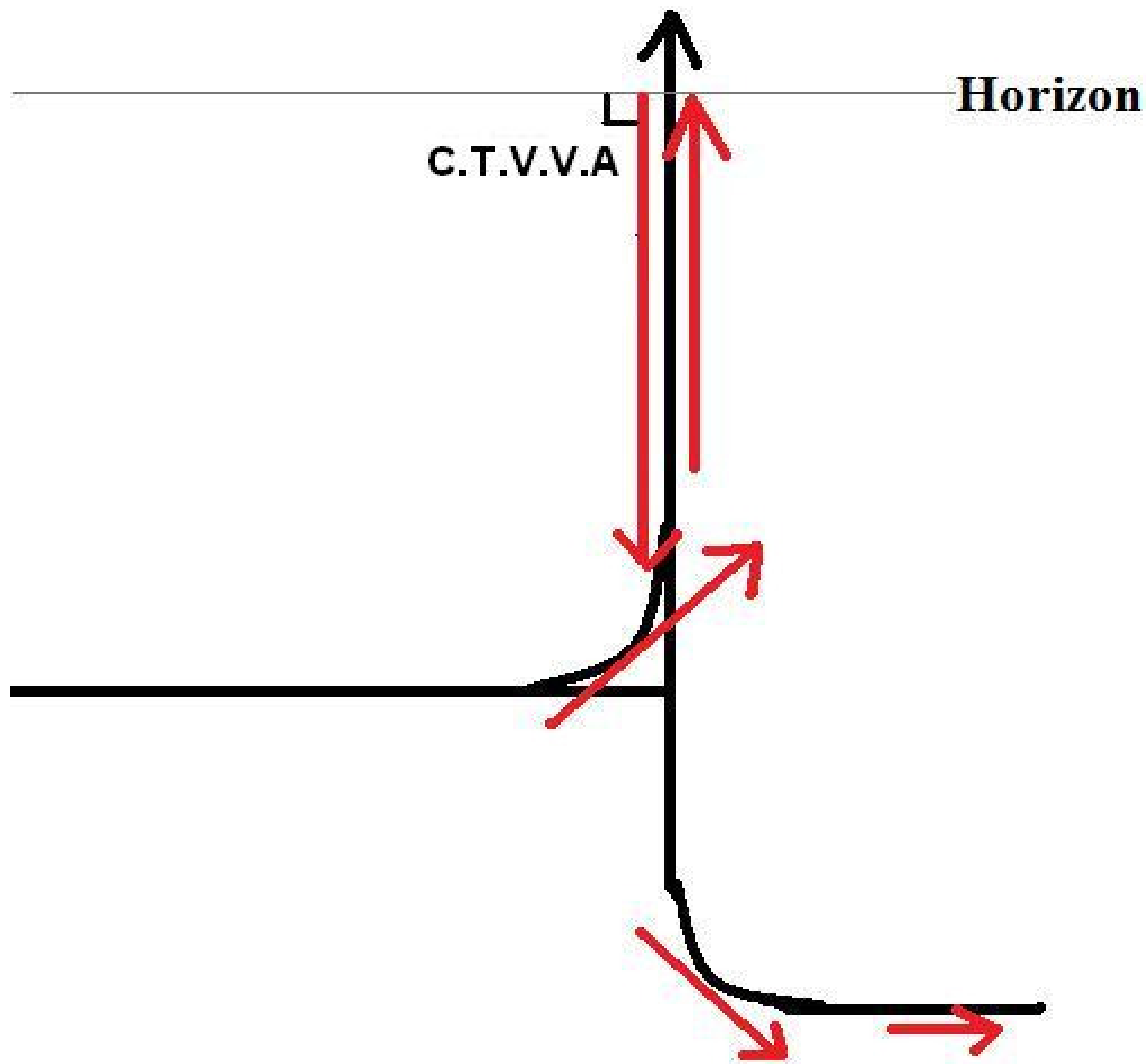
Loop

After a period of level flight, the yoke comes back to pitch the aircraft and to continue to pitch the aircraft up, the aircraft continues to pitch to exit the manoeuvre ideally at the same altitude and airspeed entered. The C.T.V.V.A. is 90 degrees.



Hammer Head

After a period of level flight, the yoke comes back sharply to pitch the aircraft perfectly vertical. Once the airspeed is low enough, or even 0, the left rudder is applied hard, to yaw the aircraft 180 degrees left – perfectly vertical again, but facing down. The yoke is pulled back sharply again, until the aircraft has achieved level flight. The C.T.V.V.A. is 90 degrees, after the pivot at the top.

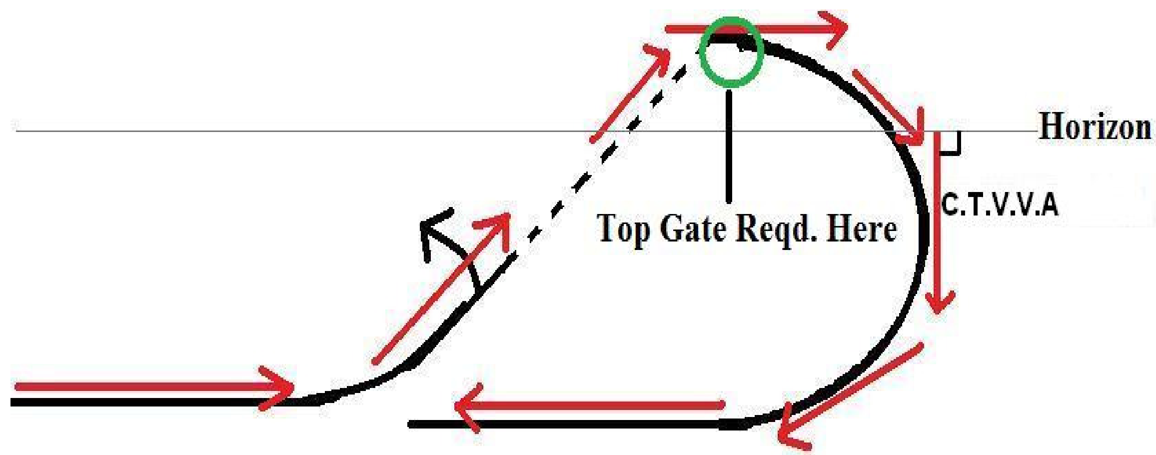


Very Dangerous

Very dangerous manoeuvres are classified as such, similar to the moderately dangerous manoeuvres, due to the 90 degree critical tangential velocity vector angle. What separates these from the moderately dangerous manoeuvres is that a moderately dangerous manoeuvre has a "first half", which naturally converts the aircrafts airspeed to altitude (kinetic to potential) required to complete the second half of the manoeuvre. Very dangerous manoeuvres do not, and require the pilot to manually position the aircraft with a certain airspeed and altitude, needed to complete the manoeuvre, without hitting the ground, which almost always result in the complete destruction of the aircraft and the death of the pilot. This certain airspeed and altitude is known as a "**top gate**".

Half Reverse Cuban Eight

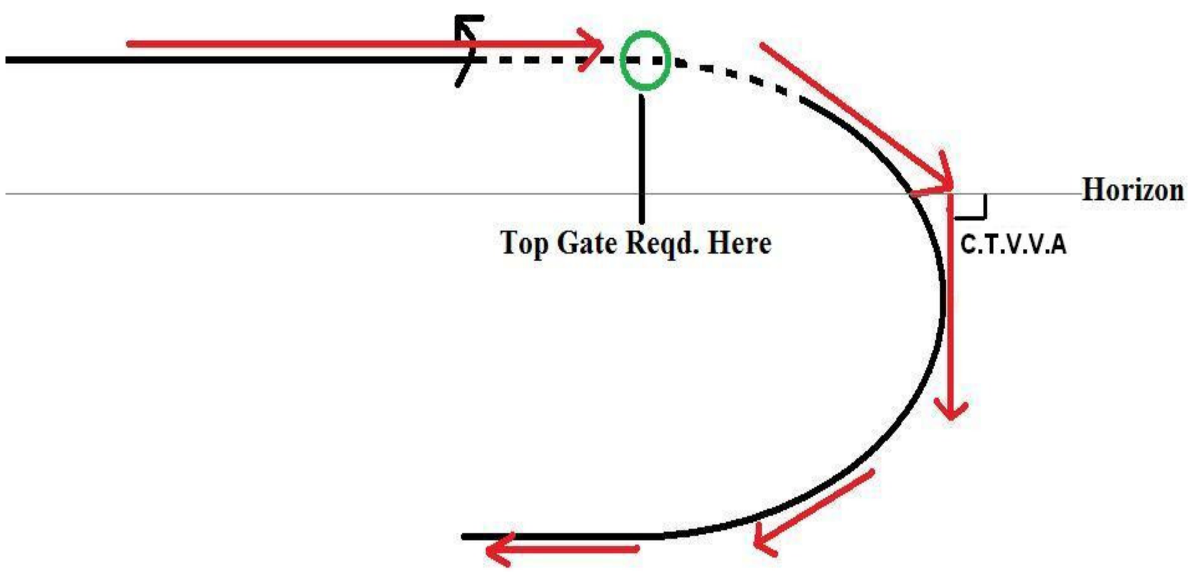
After a period of level flight, the yoke is sharply pulled back, then after a period of ascent, a half roll occurs to induce **inverted** ascent, then after a top gate is met, the yoke is pulled back (identical to the second half of a loop) to exit the manoeuvre in level flight. The C.T.V.V.A. is 90 degrees.



Note: At an air show on Sept. 14, 2004 at Mountain Home Air Force Base, Idaho, a pilot of the United States Thunder Birds ejected only 0.8 seconds before his F-16 aircraft hit the ground, after not making the top gate of the Half Reverse Cuban Eight. **(See eastsidemodders.com/tbird.html)**.

Split S

After a period of level flight, a half roll to the inverted occurs, then after the top gate is met, the yoke is pulled back to exit the manoeuvre in level flight.



Top Gate

We now know that manoeuvres classified as very dangerous require a top gate, that is, a required airspeed and height are needed to perform the manoeuvre without hitting the ground. To understand the top gate, let's look at the altitude lost during a pull from a vertical downline – when the C.T.V.V.A. is 90 degrees. Let's start with the lift equation. We already know that:

$$\mathbf{F_{lift} = (K)\ (C_L)\ (v^2)}$$

It is possible to derive the following from the lift equation:

$$\mathbf{V_{stall}\ (G) = V_{stall}(1\ G)\ (\sqrt{G})}$$

That is, the speed in which the aircraft stalls at under X G's is equal to the speed in which it stalls at under 1G, multiplied by the square root of X.

Example: with a 1 G V_{stall} of 60 mph, we can pull 4 G above (60 x $\sqrt{4}$) or 120 mph. Slower than 120 mph, we will stall before we can develop 4 G's of lift. Above 120 mph, we can pull more than 4G.

An interesting application of the above is during a turn, or pullout from a vertical downline. Intuitively we might want to develop maximum lift, and this is obtained at the **stalling angle of attack** where Cl_{max} is obtained.

The formula for acceleration, vertical or horizontal is:

$$\mathbf{Acceleration = \frac{V^2}{R}} \qquad \text{(Radius)}$$

And we know that acceleration is simply G Forces, so we can rearrange the above formula to state:

$$\mathbf{R = \frac{V^2}{G}}$$

Which makes sense. With a constant velocity, as we increase G, radius decreases. And as you decrease G, radius increases. This is very crucial – for a vertical or horizontal turn, as you increase the velocity, the radius of the turn must increase by the change in velocity squared. Meaning if an aircraft going at four times the regular speed for a turn, it must have SIXTEEN times more distance between the aircraft at the beginning of the turn, and the end of the turn, assuming the aircraft is to be under the same amount of G.

> **Note: In an air show in Sarnia, Ontario in 2001, Carey Moore stalled an aircraft in a turn from being parallel to the runway (the downwind leg of the aircrafts circuit) to the runway, because his airspeed was significantly greater then what he was used to, and didn't fly the downwind farther from the runway, thereby not giving himself the required radius for the turn.**

Back to the vertical downline: with a 1 G V_{stall} of 60 mph, in a 120 mph vertical downline, we can pull a maximum of 4 G at the stall Angle of Attack as per above.

$$\mathbf{R = \frac{(120 \times 5280 / 3600)^2}{4 \times 32}} \quad \text{(120 mph to feet per second; 4 G Forces times the regular 32 feet per second squared)}$$

$$\mathbf{= 242 \text{ feet}}$$

Note that the above does not take into account the altitude being continually lost due to Earth's gravitational field.

Now, let's see what the effect of a much higher airspeed does to the radius: with a 1 G Vs of 60 mph, in a 180 mph downline, we can pull a maximum of 9 G at the stall AOA, as per the equation above.

$$R = \frac{(180 \times 5280 / 3600)^2}{9 \times 32} = 242 \text{ feet}$$

(180 mph to feet per second)

9 G Forces times the regular 32 feet per second squared

Isn't that interesting - and quite counter-intuitive! The radius of a 120 mph maximum-performance turn is perfectly equal to the radius of a 180 mph maximum-performance turn, neglecting the effect of Earth's gravity.

Why? Because we have velocity squared is on both the top and bottom of the calculation of the radius, so they cancel each other out, giving us a constant radius, as long as we are willing to operate at the stalling angle of attack, which at high airspeeds can develop enormous G loads - perhaps more than the pilot or aircraft can withstand.

There is one important detail we have omitted above, for simplicity - the effect of earth's gravity. During the pullout, we are being constantly accelerated downward at 32 feet per second squared. So, the less time we take to fly the manoeuvre, logically the less altitude will be have lost.

So the highest possible speed, with the maximum allowable structural G at Cl_{max} will result in minimum loss of altitude from a vertical downline.

From the above, we can conclude that the altitude parameter of the top gate is crucial. If the required altitude is not achieved, the aircraft will hit the ground, regardless of how the manoeuvre is flown (airspeed, G, etc).

The airspeed parameter of the top gate determines how much G will be pulled in the manoeuvre, To develop maximum lift, the manoeuvre should be flown at C_L max, which we already know is at the stalling AOA of the wing. And, the faster the manoeuvre is flown, the less altitude is lost (remember that Earth's gravitational force is 32 feet per second squared). But care must be taken to not exceed the maximum allowable airframe structural G.

Conclusion

As we have just learned, an aircraft flying low level aerobatics is one of many great applications of physics. Using physics we have just learned about:

- energy gaining vs energy losing aerobatic manoeuvres;
- the 4 Forces acting on an aircraft, and calculating the net force upon the aircraft;
- applications of tangential velocity vectors to aircraft, and using them to classify various manoeuvres' danger;
- the top gate required at the beginning of very dangerous manoeuvres.

which are some of the aspects needed to generate an aerobatic sequence.

Bibliography

http://en.wikipedia.org/wiki/Drag_coefficient

http://www.grc.nasa.gov/WWW/K-12/airplane/dragco.html

http://www.grc.nasa.gov/WWW/K-12/airplane/forces.html

http://www.aerospaceweb.org/question/airfoils/q0150b.shtml

http://www.aerospaceweb.org/question/airfoils/q0150b.shtml

http://en.wikipedia.org/wiki/Centripetal_force#Geometric_derivation

http://www.ac.wwu.edu/~vawter/PhysicsNet/Topics/RotationalKinematics/TangentialVelocity.html

http://www.centennialofflight.gov/essay/Theories_of_Flight/drag/TH4.htm

Senior Physics Notes, Revised. Coles, 1979. Page 58

Flight Training, Smiths Falls Airport, June 3, 2005 – present

Aircraft Engine Design by Jack D. Mattingly, William H. Heiser, and Davit T. Pratt, Appendix E :

http://books.google.ca/books?id=2Wy5rpdm3DMC&pg=PA537&lpg=PA537&dq=how+to+measure+aircrafts+thrust&source=web&ots=5c-Ol0hcS6&sig=5eawRBLekQBouL9VBJvNOhXSBlI&hl=en&sa=X&oi=book_result&resnum=10&ct=result#PPA537,M1